

Gravity Induced Wave Function Collapse

G. Gasbarri,^{1,2,3} M. Toroš,^{2,3} S. Donadi,^{2,3} and A. Bassi^{2,3}

¹*Abdus Salam ICTP, Strada Costiera 11, I-34151 Trieste, Italy*

²*Department of Physics, University of Trieste, Strada Costiera 11, 34151 Trieste, Italy*

³*Istituto Nazionale di Fisica Nucleare, Trieste Section, Via Valerio 2, 34127 Trieste, Italy*

Starting from an idea of S.L. Adler [1], we develop a novel model of gravity-induced spontaneous wave-function collapse. The collapse is driven by complex stochastic fluctuations of the spacetime metric. After having derived the fundamental equations, we prove the collapse and amplification mechanism, the two most important features of a consistent collapse model. Under reasonable simplifying assumptions, we constrain the strength ξ of the complex metric fluctuations with available experimental data. We show that $\xi \geq 10^{-26}$ in order for the model to guarantee classicality of macro-objects, and at the same time $\xi \leq 10^{-20}$ in order not to contradict experimental evidence. As a comparison, in the recent discovery of gravitational waves in the frequency range 35 to 250 Hz, the (real) metric fluctuations reached a peak $\xi \sim 10^{-21}$.

I. INTRODUCTION

The possibility for quantum mechanics to be the limiting case of an underlying nonlinear theory has been often considered in the literature [2–7]. A straightforward motivation is that linear models typically are an approximation of nonlinear ones [6]. A stronger motivation is that they open the way to solving the quantum measurement problem [8]. In this latter context, models of spontaneous wave function collapse [9–12] provide a consistent phenomenology describing the collapse of the wave function during a measurement, via extra nonlinear and stochastic terms added to the dynamics. Due to their intrinsic nonlinearity, these models also offer a way out to some of the puzzles in quantum gravity and cosmology [13–15].

The common feature of all collapse models is a classical noise, coupled nonlinearly to the quantum wave function. The typical collapse equation, in the Itô form, is:

$$d\psi_t = \left[-\frac{i}{\hbar} \hat{H}_0 dt + \sqrt{\lambda} \sum_j (\hat{A}_j - \langle \hat{A}_j \rangle_t) dW_{j,t} - \frac{\lambda}{2} \sum_j (\hat{A}_j - \langle \hat{A}_j \rangle_t)^2 dt \right] \psi_t, \quad (1)$$

where \hat{H} is the standard quantum Hamiltonian, $\{\hat{A}_j\}_j$ a set of self-adjoint commuting operators, $\langle \hat{A}_j \rangle_t = \langle \psi_t | \hat{A}_j | \psi_t \rangle$ and $W_{j,t}$ a set of independent Wiener processes, which force the wave function to collapse towards one of the common eigenstates of the operators \hat{A}_j [16]. The positive coupling constant λ sets the strength of the collapse mechanism.

Eq. (1) should be considered as a phenomenological equation, raising the question why it takes that form. A justification comes from the following argument first proposed by Adler [7]. Consider the Hamiltonian:

$$\hat{H} = \hat{H}_0 + i\hbar\sqrt{\lambda} \sum_j \hat{A}_j w_{j,t}, \quad (2)$$

where $w_{j,t} = dW_{j,t}/dt$ is a set of independent white noises. It describes the coupling of a quantum system with a set of external classical noises, through the operators \hat{A}_j . It is a reasonable phenomenological ansatz, except for the fact the second term is anti-hermitian. As a consequence, the norm of ψ_t is not conserved, jeopardizing the physical meaning of the wave function. The obvious thing to do is to replace ψ_t with $\psi_t/\|\psi_t\|$, but this brings in a serious problem: the resulting

equation is nonlinear and also the stochastic ensemble of states evolves nonlinearly, even in the average. This leads to superluminal signaling [17]. The problem can be avoided if one adds extra terms in Eq. (2), such that the master equation for density matrix $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$ associated to the ensemble becomes linear (and of the Lindblad type [18–20]). These new terms are precisely those, which lead to Eq. (1). Appendix A contains the derivation of what outlined here.

In the sense explained here above, the requirements of norm conservation and no-superluminal signaling added to Eq. (2), give the desired collapse equation. Likely, a sensible nonlinear pre-quantum theory, which leads to a dynamics for the wave function at the phenomenological level, will naturally embody both requirements. The open issue now, is how to justify \hat{H} in (2), in particular why the coupling should be anti-hermitian, and what is the suitable choice for the operators \hat{A}_j , which select the basis along which the collapse occurs. While there is no answer to the first question, more than the hope that the pre-quantum theory will provide a natural answer, one can say more about the second question.

Quite often the literature suggest that the collapse is driven by gravity [21–28]. This is the only hope one can have, to link the collapse to a known force, since all other forces as we know them have been successfully quantized, therefore they cannot provide the anti-hermitian coupling needed for the non-linear collapse. But there is a stronger motivation. The collapse scales with the mass/size of the system [9, 10], and localizes the wave function in space. Then, the natural candidate for the operators \hat{A}_j is the *local mass density* $\hat{m}(\mathbf{x}) = \sum_i m_i \delta^{(3)}(\mathbf{x} - \hat{\mathbf{x}}_i)$, coupled to a noise $w(\mathbf{x}, t)$ spread through space [43]:

$$\hat{H} = \hat{H}_0 + i\hbar\sqrt{\xi} \int d^3x \hat{m}(\mathbf{x})w(\mathbf{x}, t). \quad (3)$$

A random *gravitational* field naturally provides such a coupling (see Appendix B), which would contain an anti-hermitian part if the field has an imaginary component. In [1] arguments are presented, as to why the metric could be classical and complex-valued. Following this idea we will explore the consequences of assuming a *complex non-white classical noise coupled to the local mass density*.

The paper is organized as follows. In Section II we derive, to the first meaningful perturbative order, the general collapse equation for the wave function, as well as the associated master equation, in the case of N complex valued coloured random noises $h_i(t)$, each coupled to an operator \hat{A}_i . The literature so far considered only the case of real valued coloured noises [12]. In Section III we show the collapse mechanism. In Section IV we consider specifically a noise field $w(\mathbf{x}, t)$ coupled to the local mass density $\hat{M}(\mathbf{x})$ and discuss the amplification mechanism, one of the crucial properties of any collapse model. In Section V we discuss the bounds on the spectrum of the noise, which are set by current experiments. We conclude the paper with a discussion of the results (Section VI).

II. MASTER AND COLLAPSE EQUATIONS

We have seen how the idea of a complex gravitational stochastic background inducing the collapse of the wave function leads to a collapse model where the noise is *complex* valued and in general *coloured*. Since this has not been discussed in the literature so far, in this section we derive the appropriate collapse equation and the master equation, following the same strategy as in Appendix A for a real valued white noise.

The starting point is the following generalized Schrödinger equation

$$i\hbar\partial_t |\phi_t\rangle = \left[\hat{H}_0 + \xi \sum_{i=1}^N \hat{A}_i h_i(t) + \hat{O} \right] |\phi_t\rangle \quad (4)$$

where \hat{A}_i are arbitrary self-adjoint operators and $h_i(t)$ are N complex Gaussian noises, with zero average and correlation function:

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}}[h_i^*(t)h_j(\tau)] &= D_{ij}(t, \tau), \\ \mathbb{E}_{\mathbb{Q}}[h_i(t)h_j(\tau)] &= S_{ij}(t, \tau).\end{aligned}\tag{5}$$

$D_{ij}(t, \tau)$ and $S_{ij}(t, \tau)$ are complex functions with magnitude of order 1 and \hat{O} is an operator yet to be defined. The parameter ξ sets the strength of the noise, which is assumed to be small. Following the scheme outlined in the introduction, we will determine \hat{O} by the requirement of no-faster-than-light signaling.

Since the norm of $|\phi_t\rangle$ is not conserved, we consider the normalized state $|\psi_t\rangle = |\phi_t\rangle / \|\phi_t\|$, which solves the equation

$$\begin{aligned}i\hbar\partial_t|\psi_t\rangle &= \left[\hat{H}_0 + \xi \sum_{i=1}^N \left(\hat{A}_i h_i(t) - i\langle A_i \rangle_t h_i^I(t) \right) + \hat{O} - \frac{1}{2}\langle \hat{O} - \hat{O}^\dagger \rangle_t \right] |\psi_t\rangle \\ &= \left(\hat{H}_t - \frac{1}{2}\langle \hat{H}_t - \hat{H}_t^\dagger \rangle_t \right) |\psi_t\rangle\end{aligned}\tag{6}$$

with

$$\hat{H}_t = \hat{H}_0 + \xi \sum_{i=1}^N \hat{A}_i h_i(t) + \hat{O}.\tag{7}$$

As expected, the normalized vector evolves according to a nonlinear stochastic dynamics. The stochastic ensemble of pure states $\rho_t^h = |\psi_t\rangle\langle\psi_t|$ obeys the following dynamics:

$$i\hbar\partial_t\rho_t^h = \left[\hat{H}_0 + \xi \sum_{i=1}^N \left(\hat{A}_i h_i(t) - i\langle A_i \rangle_t h_i^I(t) \right) + \hat{O} - \frac{1}{2}\langle \hat{O} - \hat{O}^\dagger \rangle_t \right] \rho_t^h - h.c.\tag{8}$$

Taking the expectation value to compute the dynamics for the density matrix $\rho_t = \mathbb{E}[\rho_t^h]$, one obtains in general a non linear evolution for the ensemble, which implies the possibility of faster than light signaling [17]. This can be avoided with a proper choice of the operator \hat{O} . Contrary to the white-noise case, identifying the correct form of \hat{O} is very difficult (in general, impossible) since the dependence of the right-hand-side of the above equation on the noise h is highly nontrivial. This means that one is not able to compute the stochastic average and without such knowledge, \hat{O} cannot be determined. A way to circumvent the problem is to proceed perturbatively [29]. We Taylor-expand ρ_t^h in terms of ξ :

$$\rho_t^h = \rho_{0,t}^h + \xi\rho_{1,t}^h + \xi^2\rho_{2,t}^h + \mathcal{O}(\xi^3)\tag{9}$$

where, for $t = 0$, all terms except the first one are zero. We expand also \hat{O} in powers of ξ [44]:

$$\hat{O} = \xi\hat{O}_1 + \xi^2\hat{O}_2 + \mathcal{O}(\xi^3).\tag{10}$$

The perturbation expansion of Eq. (8) gives the following system of equations

$$\begin{aligned}
i\hbar\partial_t\rho_{0,t}^h &= \hat{H}_0\rho_{0,t}^h - h.c. \\
i\hbar\partial_t\rho_{1,t}^h &= \hat{H}_0\rho_{1,t}^h + \left(\sum_{i=1}^N (\hat{A}_i h_i(t) - i\langle A_i \rangle_t^0 h_i^I(t) + \hat{O}_1 + \frac{1}{2}\langle \hat{O}_1 - \hat{O}_1^\dagger \rangle_t^0) \right) \rho_{0,t}^h - h.c. \\
i\hbar\partial_t\rho_{2,t}^h &= \hat{H}_0\rho_{2,t}^h + \left(\sum_{i=1}^N (\hat{A}_i h_i(t) - i\langle A_i \rangle_t^0 h_i^I(t) + \hat{O}_1 - \frac{1}{2}\langle \hat{O}_1 - \hat{O}_1^\dagger \rangle_t^0) \right) \rho_{1,t}^h \\
&\quad - \left(\sum_{i=1}^N i\langle A_i \rangle_t^1 h_i^I(t) - \hat{O}_2 + \frac{1}{2}\langle \hat{O}_1 - \hat{O}_1^\dagger \rangle_t^1 + \frac{1}{2}\langle \hat{O}_2 - \hat{O}_2^\dagger \rangle_t^0 \right) \rho_{0,t}^h - h.c.
\end{aligned} \tag{11}$$

where $\langle A \rangle_t^n = \text{Tr}(\hat{A}\rho_{n,t}^h)$, and similarly for the other operators. We can formally solve the above system of equations:

$$\begin{aligned}
\rho_{0,t}^h &= e^{i\hat{H}_0 t} \rho_0 e^{-i\hat{H}_0 t} \\
\rho_{1,t}^h &= -\frac{i}{\hbar} \sum_{i=1}^N \int_0^t d\tau \left(\hat{A}_i(\tau-t) h_i(\tau) - i\langle A_i \rangle_\tau^0 h_i^I(\tau) + \hat{O}_1(\tau-t) - \frac{1}{2}\langle O_1 - O_1^\dagger \rangle_\tau^0 \right) \rho_{0,t}^h + h.c. \\
\rho_{2,t}^h &= -\frac{i}{\hbar} \sum_{i=1}^N \int_0^t d\tau \left(\hat{A}_i(\tau-t) h_i(\tau) - i\langle A_i \rangle_\tau^0 h_i^I(\tau) + \hat{O}_1(\tau-t) - \frac{1}{2}\langle O_1 - O_1^\dagger \rangle_\tau^0 \right) e^{i\hat{H}(t-\tau)} \rho_{1,\tau}^h e^{-i\hat{H}(t-\tau)} \\
&\quad - \frac{i}{\hbar} \sum_{i=1}^N \int_0^t d\tau \left(i\langle A_i \rangle_\tau^1 h_i^I(\tau) - \hat{O}_2(\tau-t) + \frac{1}{2}\langle \hat{O}_1 - \hat{O}_1^\dagger \rangle_\tau^1 + \frac{1}{2}\langle O_2 - O_2^\dagger \rangle_\tau^0 \right) \rho_{0,t}^h + h.c.
\end{aligned} \tag{12}$$

where $\hat{A}_i(t)$ is the operator \hat{A}_i in the interaction picture at time t :

$$\hat{A}_i(t) = e^{\frac{i}{\hbar}\hat{H}_0 t} \hat{A}_i e^{-\frac{i}{\hbar}\hat{H}_0 t}, \tag{13}$$

and similarly for the operator \hat{O} .

Now we are in the position to compute a closed equation for the averaged density matrix $\mathbb{E}[\rho_t^h]$. We plug the solutions in Eq. (12) into Eq. (9); in this way the stochasticity is entirely contained in polynomials of h , whose correlations are known. We can then explicitly compute the stochastic average of each term. Collecting all pieces together, we arrive at the following perturbative equations for the ensemble, which is valid up to order ξ^2 .

$$\begin{aligned}
i\hbar\partial_t\rho_{0,t} &= \hat{H}_0\rho_{0,t} - h.c. \\
i\hbar\partial_t\rho_{1,t} &= \hat{O}_1\rho_{0,t} + \frac{1}{2}\langle O_1 - O_1^\dagger \rangle_t^0 \rho_{0,t} - h.c. \\
i\hbar\partial_t\rho_{2,t} &= -\frac{i}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau S_{ij}(t,\tau) (\hat{A}_i - \langle A_i \rangle_t^0) (\hat{A}_j(\tau-t) - \langle A_j(\tau-t) \rangle_t^0) \rho_{0,t} \\
&\quad + \frac{i}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau D_{ij}(t,\tau) (\hat{A}_j(\tau-t) - \langle A_j(\tau-t) \rangle_t^0) \rho_{0,t}(t,\tau) (\hat{A}_i - \langle A_i \rangle_t^0) \\
&\quad + \frac{i}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau (S_{ij}(t,\tau) - D_{ij}(t,\tau)) \langle A_i (A_j(\tau-t) - \langle A_j(\tau-t) \rangle_t^0) \rangle_t^0 \rho_{0,t} \\
&\quad + \hat{O}_2 + \frac{1}{2}\langle O_2 - O_2^\dagger \rangle_t^0 \rho_{0,t} - h.c.
\end{aligned} \tag{14}$$

The above equations are again non-linear. The non-linear terms can be removed by choosing

$$\begin{aligned}\hat{O}_1 &= 0 \\ \hat{O}_2 &= + \frac{i}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau (S_{ij}(t, \tau) - D_{ij}(t, \tau)) \hat{A}_i (\hat{A}_j(t - \tau) - \langle A_j(t - \tau) \rangle_t^0) \\ &\quad - \frac{i}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau (S_{ij}(t, \tau) - D_{ij}^*(t, \tau)) \langle A_i \rangle_t^0 \hat{A}_j(\tau - t)\end{aligned}\quad (15)$$

Substituting this expression in Eq. (14) and re-summing the Taylor series one arrives at:

$$\partial_t \rho_t = - \frac{i}{\hbar} \hat{H}_0 \rho_t - \frac{\xi^2}{\hbar^2} \int_0^t d\tau \sum_{i,j=1}^N D_{ij}(t, \tau) \left(\hat{A}_i \hat{A}_j(\tau - t) \rho_t - \hat{A}_j(\tau - t) \rho_t \hat{A}_i \right) + \mathcal{O}(\xi^3) + h.c. \quad (16)$$

or equivalently:

$$\begin{aligned}\partial_t \rho_t &= - \frac{i}{\hbar} [\hat{H}_0, \rho_t] - \frac{\xi^2}{\hbar^2} \sum_{i,j=1}^N \int_0^t d\tau D_{ij}^R(t, \tau) [\hat{A}_i, [\hat{A}_j(\tau - t), \rho_t]] + \\ &\quad - \frac{i\xi^2}{\hbar^2} \sum_{i,j=1}^N \int_0^t d\tau D_{ij}^I(t, \tau) [\hat{A}_i, \{ \hat{A}_j(\tau - t), \rho_t \}] + \mathcal{O}(\xi^3),\end{aligned}\quad (17)$$

where the superscript R/I stands for the real/imaginary part. Note that the correlator $S_{ij}(t, \tau)$ does not appear. The collapse equation for the wave function turns out to be, up to second order in ξ^2 :

$$\begin{aligned}i\hbar \partial_t |\psi_t\rangle &= \left[\hat{H}_0 + \xi \left(\sum_{i=1}^N (\hat{A}_i h_i(t) - i \langle A_i \rangle_t h_i^I(t)) \right) \right. \\ &\quad + \frac{i\xi^2}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau (S_{ij}(t, \tau) - D_{ij}(t, \tau)) \hat{A}_i (\hat{A}_j(t - \tau) - \langle A_j(t - \tau) \rangle_t) \\ &\quad - \frac{i\xi^2}{\hbar} \sum_{i,j=1}^N \int_0^t d\tau (S_{ij}(t, \tau) - D_{ij}^*(t, \tau)) \langle A_i \rangle_t \hat{A}_j(\tau - t) \\ &\quad - \frac{i\xi^2}{2\hbar} \sum_{i,j=1}^N \int d\tau (S_{ij}(t, \tau) - D_{ij}(t, \tau)) (\langle A_i A_j(\tau - t) \rangle_t - 2 \langle A_i \rangle_t \langle A_j(\tau - t) \rangle_t) \\ &\quad \left. - \frac{i\xi^2}{2\hbar} \sum_{i,j=1}^N \int d\tau (S_{ij}^*(t, \tau) - D_{ij}^*(t, \tau)) (\langle A_j(\tau - t) A_i \rangle_t - 2 \langle A_i \rangle_t \langle A_j(\tau - t) \rangle_t) \right] |\psi_t\rangle. \quad (18)\end{aligned}$$

Performing the Markovian limit by imposing $D_{ij}(t, s) = \delta(t - s) \tilde{D}_{ij}(t)$ and $S_{ij}(t, s) = \delta(t - s) \tilde{S}_{ij}(t)$

one ends up with the following stochastic Schrödinger equation in the Stratonovich form:

$$\begin{aligned}
i\hbar\partial_t|\psi_t\rangle = & \left[\hat{H}_0 + \xi \left(\sum_{i=1}^N (\hat{A}_i h_i(t) - i\langle A_i \rangle_t h_i^I(t)) \right) \right. \\
& + \frac{i\xi^2}{\hbar} \sum_{i,j=1}^n (\tilde{S}_{ij}(t) - \tilde{D}_{ij}(t)) [(\hat{A}_i - \langle A_i \rangle_t)(\hat{A}_j - \langle A_j \rangle_t) + \frac{1}{2}(\langle A_i A_j \rangle_t + \langle A_j A_i \rangle_t - 2\langle A \rangle_i \langle A \rangle_j)] \\
& \left. - \frac{i\xi^2}{\hbar} \sum_{i,j=1}^N (\tilde{S}_{ij}^I(t) - \tilde{D}_{ij}^I(t)) (\langle A_i A_j \rangle_t - 2\langle A_j A_i \rangle_t) + \frac{i2\xi}{\hbar} \sum_{i,j=1}^n \tilde{D}_{ij}^I(t) \langle A_i \rangle_t \hat{A}_j \right] |\psi_t\rangle. \quad (19)
\end{aligned}$$

This equation is a generalization of Eq. (7.43) in [11]. The first two lines correspond to Eq. (7.43), with the replacement $\gamma \rightarrow \tilde{S}_{ij}(t) - \tilde{D}_{ij}(t)$, taking also into account that in our case the operators A_i are not assumed to commute; the third line is associated to the complex part of the noise, while in [11] the noise was assumed to be real.

In the next sections we will discuss the main consequences of Eqs. (17) and (18): the collapse of the wave function, the presence, under certain conditions, of an amplification mechanism and the experimental predictions.

III. COLLAPSE OF THE WAVE FUNCTION

We now establish under which conditions the dynamics given by Eq. (18), when $H_0 = 0$, induces the collapse of the state vector $|\psi\rangle_t$ into one of the eigenstate of \hat{A}_i , assuming that these operators commute with each other and therefore have a common set of eigenstates. We will follow the procedure outlined in Sec. IIa of [29]. We neglect the standard evolution since we are focusing on the collapse process. This approximation, in general not true, is good for macroscopic objects. In fact, given the amplification mechanism, which we will describe in the next section, the effect of the collapse increases with the mass of the system, making it dominant with respect to the standard evolution for large objects.

We consider the stochastic average of the variance $V_A(t) = \langle \hat{A}^2 \rangle_t - \langle \hat{A} \rangle_t^2$ of an operator \hat{A} which commutes with each \hat{A}_i . One may prove that, for any n :

$$\mathbb{E}[\langle \hat{A}^n \rangle_t] = \text{Tr}[\rho_t \hat{A}^n] = \text{Tr}[\rho_0 \hat{A}^n] = \mathbb{E}[\langle \hat{A}^n \rangle_0]. \quad (20)$$

Then, exploiting the perturbation series in Eq. (12) and performing the stochastic average one can obtain:

$$\begin{aligned}
\mathbb{E}[\langle A \rangle_t^2] &= \mathbb{E}[\langle A \rangle_0] \\
&- \frac{2\xi^2}{\hbar^2} \sum_{i,j=1}^N \int_0^t d\tau \int_0^\tau ds \left(S_{ij}^R(t, \tau) - D_{ij}^R(t, \tau) \right) \langle \langle A \rangle_0 (A_i - \langle A_i \rangle_0)_0 \rangle \langle \langle A \rangle_0 (A_j - \langle A_j \rangle_0)_0 \rangle + \mathcal{O}(\xi^3). \quad (21)
\end{aligned}$$

Given the above result, one can now compute the stochastic average of the variance $V_A(t)$, arriving at:

$$\mathbb{E}[V_A(t)] = V_A(0) - \frac{2\xi^2}{\hbar^2} \sum_{i,j=1}^N \int_0^t d\tau F_{ij}(\tau) \left(\langle A A_i \rangle_0 - \langle A \rangle_0 \langle A_i \rangle_0 \right) \left(\langle A A_j \rangle_0 - \langle A \rangle_0 \langle A_j \rangle_0 \right) + \mathcal{O}(\xi^3) \quad (22)$$

where:

$$F_{ij}(\tau) = \int_0^\tau ds (D_{ij}^R(\tau, s) - S_{ij}^R(\tau, s)). \quad (23)$$

According to [45] the positivity of $F(\mathbf{x}, \mathbf{y}, \tau)$ in the limit $t \rightarrow \infty$ is a sufficient condition to guarantee the reduction properties of Eq. (18). In fact, whenever F is non negative, Eq. (22) implies that, for large times $\langle A A_i \rangle_\tau - \langle A \rangle_\tau \langle A_i \rangle_\tau$ converges to 0 for any realization of the noise, with the only possible exception of a subset of measure 0. In particular, when \hat{A} is equal to A_i we have

$$\lim_{t \rightarrow \infty} \langle A_i A_i \rangle_\tau - \langle A_i \rangle_\tau \langle A_i \rangle_\tau = \lim_{t \rightarrow \infty} V_{A_i}(t) = 0. \quad (24)$$

This means that any initial state converges asymptotically, with probability 1, to one of the eigenstates of the operator \hat{A}_i .

A related question is how fast the wave function collapses. The decoherence rate of the associated master Eq. (17) provides a good measure. If we set $\hat{H}_0 = 0$, we immediately obtain the decoherence rate in the basis of the common eigenstates of the operators \hat{A}_i :

$$\begin{aligned} \rho_t(\alpha, \beta) &= \exp \left(-\frac{\xi^2}{\hbar^2} \sum_{i,j=1}^N \int_0^t d\tau \int_0^\tau ds D_{ij}^R(\tau, s) (\alpha_i \alpha_j - \alpha_i \beta_j - \alpha_j \beta_i + \beta_j \beta_i) \right. \\ &\quad \left. + i D_{ij}^I(\tau, s) (\alpha_i \alpha_j + \alpha_i \beta_j + \alpha_j \beta_i + \beta_j \beta_i) \right) \rho_0(\alpha, \beta) \\ &= \exp \left(-\frac{\xi^2}{\hbar^2} \sum_{i,j=1}^N \int_0^t d\tau \int_0^\tau ds D_{ij}(\tau, s) (\alpha_i \alpha_j - \beta_i \beta_j) - D_{i,j}^*(\tau, s) (\alpha_i \beta_j + \alpha_j \beta_i) \right) \rho_0(\alpha, \beta) \end{aligned} \quad (25)$$

where $\rho_t(\alpha, \beta) = \langle \alpha | \rho_t | \beta \rangle$ and $|\alpha\rangle$ ($|\beta\rangle$) is one element of the basis, *i.e.* $\hat{A}_i |\alpha\rangle = \alpha_i |\alpha\rangle$. It is worth studying the case where there is only one collapse operator and the correlation is real and delta correlated in time, *i.e.*

$$D(\tau, s) = \tau_0 \delta(\tau - s) \quad (26)$$

with τ_0 a real parameter with the dimensions of a time. Then Eq. (25) reduces to:

$$\rho_t(\alpha, \beta) = e^{-\frac{\xi^2 \tau_0 t}{\hbar^2} (\alpha - \beta)^2} \rho_0(\alpha, \beta) \quad (27)$$

where the decoherence rate is constant in time and is determined by $\tau_0 \xi^2$.

IV. MASTER EQUATION FOR THE CENTER OF MASS AND THE AMPLIFICATION MECHANISM

After the collapse of the wave function, the next fundamental requirement for a good collapse model is the amplification mechanism: the center of mass wave function of a composite system should collapse with a rate which increases with the size of the system. This is necessary in order for the equation to preserve the quantum properties of microscopic systems and, at the same time, to guarantee the classical properties of macroscopic objects.

Instead of considering the problem in full generality as done in the previous two sections, we focus our analysis to the case of interest here: the collapse noise coupled to the mass density operator $\hat{m}(\mathbf{x})$. In this case Eq. (17) takes the form:

$$\begin{aligned} \partial_t \rho_t = & -\frac{i}{\hbar} [\hat{H}_0, \rho_t] + \\ & -\frac{\xi^2 c^4}{\hbar^2} \int d\mathbf{x} \int d\mathbf{y} \int_0^t d\tau D^R(\mathbf{x} - \mathbf{y}, t - \tau) [\hat{m}(\mathbf{x}), [\hat{m}(\mathbf{y}, \tau - t), \rho_t]] + \\ & -\frac{i\xi^2 c^4}{\hbar^2} \int d\mathbf{x} \int d\mathbf{y} \int_0^t d\tau D^I(\mathbf{x} - \mathbf{y}, t - \tau) [\hat{m}(\mathbf{x}), \{\hat{m}(\mathbf{y}, \tau - t), \rho_t\}], \end{aligned} \quad (28)$$

where D^R and D^I are the real and the imaginary parts of the correlation function of the noise field [46] $D(\mathbf{x}, \mathbf{y}; t, \tau) = \mathbb{E}[h^*(\mathbf{x}, t)h(\mathbf{y}, \tau)]$. In writing the above equation, we assumed that the noise is statistically homogeneous over space and time: $D^{R,I}(\mathbf{x}, \mathbf{y}, t, \tau) = D^{R,I}(\mathbf{x} - \mathbf{y}, t - \tau)$. We consider a system of N point-like particles. The mass density function is:

$$\hat{m}(\mathbf{x}) = \sum_{i=1}^N m_i \delta(\mathbf{x} - \hat{\mathbf{x}}_i) = \sum_{i=1}^N \frac{m_i}{(2\pi\hbar)^3} \int d\mathbf{Q} e^{\frac{i}{\hbar} \mathbf{Q} \cdot (\mathbf{x} - \hat{\mathbf{x}}_i)}. \quad (29)$$

Substituting Eq. (29) into Eq. (28) and performing the integration over \mathbf{x} and \mathbf{y} we arrive at the expression:

$$\begin{aligned} \partial_t \rho_t = & -\frac{i}{\hbar} [\hat{H}_0, \rho_t] + \\ & -\frac{\xi^2 c^4}{\hbar^2} \sum_{i,j=1}^N \frac{m_i m_j}{(2\pi\hbar)^3} \int_0^t d\tau \int d\mathbf{Q} \tilde{D}^R(\mathbf{Q}, t - \tau) \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i}, \left[e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)}, \rho_t \right] \right] + \\ & -\frac{i\xi^2 c^4}{\hbar^2} \sum_{i,j=1}^N \frac{m_i m_j}{(2\pi\hbar)^3} \int_0^t d\tau \int d\mathbf{Q} \tilde{D}^I(\mathbf{Q}, t - \tau) \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i}, \left\{ e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)}, \rho_t \right\} \right] \end{aligned} \quad (30)$$

where we defined:

$$\tilde{D}^\beta(\mathbf{Q}, t - \tau) := \int d\mathbf{r} D^\beta(\mathbf{r}, t - \tau) e^{\frac{i}{\hbar} \mathbf{Q} \cdot \mathbf{r}} \quad \text{with} \quad \beta = R, I. \quad (31)$$

We are interested in describing the dynamics of the center of mass of the composite system. In particular, we have in mind the case of a rigid body. We introduce the center of mass coordinates:

$$\hat{\mathbf{X}} = \sum_{i=1}^N \frac{m_i}{M} \hat{\mathbf{x}}_i, \quad \hat{\mathbf{P}} = \sum_{i=1}^N \hat{\mathbf{q}}_i, \quad (32)$$

$$(33)$$

and the relative coordinates

$$\begin{cases} \hat{\mathbf{r}}_i = \hat{\mathbf{x}}_i - \hat{\mathbf{X}} & i \in (1, \dots, N-1), & \hat{\mathbf{p}}_i = \hat{\mathbf{q}}_i - \frac{m_i}{M} \hat{\mathbf{P}} & i \in (1, \dots, N-1), \\ \hat{\mathbf{r}}_N = -\sum_{i=1}^{N-1} \frac{m_i}{m_N} \hat{\mathbf{r}}_i, & & \hat{\mathbf{p}}_N = -\sum_{i=1}^{N-1} \hat{\mathbf{p}}_i, & \end{cases} \quad (34)$$

where $M = \sum_{i=1}^N m_i$ is the total mass of the system. The operators $\hat{\mathbf{r}}_N$ and $\hat{\mathbf{p}}_N$ are not independent (they are defined in terms of the other relative positions and momenta) but it is convenient to keep them to make the notation simpler. These new variables obey to the following commutation relations:

$$\begin{aligned} [\hat{\mathbf{X}}, \hat{\mathbf{P}}] &= i\hbar & [\hat{\mathbf{r}}_i, \hat{\mathbf{p}}_j] &= i\hbar \left(\delta_{ij} - \frac{m_i}{M} \right) \\ [\hat{\mathbf{X}}, \hat{\mathbf{r}}_i] &= [\hat{\mathbf{X}}, \hat{\mathbf{p}}_i] = [\hat{\mathbf{r}}_i, \hat{\mathbf{r}}_j] = [\hat{\mathbf{r}}_i, \hat{\mathbf{P}}] = 0. & i, j &\in (1, \dots, N-1) \end{aligned} \quad (35)$$

We introduce the center of mass density matrix as

$$\rho_t^{\text{CM}} := \text{Tr}_{\text{REL}}(\rho_t)$$

where $\text{Tr}_{\text{REL}}(\cdot)$ denotes the partial trace over the relative coordinates. We study the effect of the partial trace on the operators in the three lines of Eq. (30). Assuming that $\hat{H}_0 = \hat{H}_0^{\text{CM}} + \hat{H}_0^{\text{REL}}$, the term in the first line simplifies as

$$\text{Tr}_{\text{REL}}([\hat{H}_0^{\text{CM}} + \hat{H}_0^{\text{REL}}, \rho_t]) = [\hat{H}_0^{\text{CM}}, \rho_t^{\text{CM}}].$$

The double commutator in the second line can be expanded as the sum of four terms:

$$\begin{aligned} \text{Tr}_{\text{REL}} \left(\left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i}, \left[e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)}, \rho_t \right] \right] \right) &= \text{Tr}_{\text{REL}} \left(e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)} \rho_t \right) \\ &- \text{Tr}_{\text{REL}} \left(e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)} \rho_t e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} \right) - \text{Tr}_{\text{REL}} \left(e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} \rho_t e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)} \right) \\ &+ \text{Tr}_{\text{REL}} \left(\rho_t e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} \right) \end{aligned} \quad (36)$$

We consider the first term, as the calculations for the remaining terms are similar. Exploiting the commutativity of the relative and center of mass degree of freedoms, we rewrite the exponential operators in Eq. (36) as

$$e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} = e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{r}}_i}, \quad e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} = e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \hat{H}_0^{\text{REL}}(\tau-t)}, \quad (37)$$

so that

$$\text{Tr}_{\text{REL}} \left(e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)} \rho_t \right) = \quad (38)$$

$$\text{Tr}_{\text{REL}} \left(\left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{r}}_i} e^{\frac{i}{\hbar} \hat{H}_0^{\text{REL}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{r}}_j} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{REL}}(\tau-t)} \right] \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} \right] \rho_t \right).$$

We assume the motion of the relative coordinates to be a small fluctuation around the equilibrium positions \mathbf{r}_{i0} within the solid (*e.g.* in a crystalline structure), *i.e.* $\hat{\mathbf{r}}_i(t) = \mathbf{r}_{i0} + \Delta \hat{\mathbf{r}}_i(t)$, where the fluctuations $\Delta \hat{\mathbf{r}}_i(t)$ are negligible with respect to the spatial correlation length of the noise [47] within the time $t - \tau$. Under this approximation, the first square bracket in the second line of Eq. (38) becomes $e^{-\frac{i}{\hbar} \mathbf{Q} \cdot (\mathbf{r}_{i0} - \mathbf{r}_{j0})}$ and we obtain:

$$\begin{aligned} &\text{Tr}_{\text{REL}} \left(e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i} e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)} \rho_t \right) \\ &\simeq e^{-\frac{i}{\hbar} \mathbf{Q} \cdot (\mathbf{r}_{i0} - \mathbf{r}_{j0})} e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} \rho_t^{\text{CM}}, \end{aligned}$$

which depends on center of mass operators only. The other three terms can be computed in the same way and therefore we get:

$$\begin{aligned} & \text{Tr}_{\text{REL}} \left(\left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i}, \left[e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)}, \rho_t \right] \right] \right) \\ &= e^{-\frac{i}{\hbar} \mathbf{Q} \cdot (\mathbf{r}_{i0} - \mathbf{r}_{j0})} \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \left[e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)}, \rho_t^{\text{CM}} \right] \right]. \end{aligned}$$

Similarly, for the operators in the third line of Eq. (30) we obtain:

$$\begin{aligned} & \text{Tr}_{\text{REL}} \left(\left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_i}, \left\{ e^{\frac{i}{\hbar} \hat{H}_0(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{x}}_j} e^{-\frac{i}{\hbar} \hat{H}_0(\tau-t)}, \rho_t \right\} \right] \right) \\ &= e^{-\frac{i}{\hbar} \mathbf{Q} \cdot (\mathbf{r}_{i0} - \mathbf{r}_{j0})} \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \left\{ e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)}, \rho_t^{\text{CM}} \right\} \right]. \end{aligned}$$

Combining the previous results, we arrive at the following master equation for the center of mass

$$\begin{aligned} \partial_t \rho_t^{\text{CM}} &= -\frac{i}{\hbar} \left[\hat{H}_0^{\text{CM}}, \rho_t^{\text{CM}} \right] + \\ & - \frac{\xi^2 c^4}{\hbar^2} \frac{1}{(2\pi\hbar)^3} \int_0^t d\tau \int d\mathbf{Q} \tilde{D}^{\text{R}}(\mathbf{Q}, t - \tau) A(\mathbf{Q}) \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \left[e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)}, \rho_t^{\text{CM}} \right] \right] + \\ & - \frac{i\xi^2 c^4}{\hbar^2} \frac{1}{(2\pi\hbar)^3} \int_0^t d\tau \int d\mathbf{Q} \tilde{D}^{\text{I}}(\mathbf{Q}, t - \tau) A(\mathbf{Q}) \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \left\{ e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)} e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} e^{-\frac{i}{\hbar} \hat{H}_0^{\text{CM}}(\tau-t)}, \rho_t^{\text{CM}} \right\} \right] \end{aligned} \quad (39)$$

with:

$$A(\mathbf{Q}) := \sum_{i,j=1}^N m_i m_j e^{-\frac{i}{\hbar} \mathbf{Q} \cdot (\mathbf{r}_{i0} - \mathbf{r}_{j0})} = |\rho(\mathbf{Q}/\hbar)|^2 \quad (40)$$

where

$$\rho(\mathbf{k}) := \int d\mathbf{x} \rho(\mathbf{x}) e^{-i\mathbf{k}\mathbf{x}} \quad (41)$$

is the Fourier transform of the classical mass density distribution $\rho(\mathbf{x}) := \sum_{i=1}^N m_i \delta(\mathbf{x} - \mathbf{r}_i^{\text{cl}})$.

The master equation (39) for the center of mass wave function has the same structure as the single particle master equation, with the addition of the amplifying factor $A(\mathbf{Q})$, which keeps track of the fact that we are dealing with a composite object, not a point-like particle.

Typically, the noise correlators $D^{\text{R}}(\mathbf{r}, t - \tau)$ and $D^{\text{I}}(\mathbf{r}, t - \tau)$ are expected to have spatial cutoffs (the noise correlation length), respectively r_C^{R} and r_C^{I} . As for the case of the CSL model, it is interesting to study the behavior of the amplification factor in two limiting cases (for a more detailed proof of what follows, see [30]):

1. When the particles are at distances smaller than the noise correlation lengths $r_C^{\text{R}}, r_C^{\text{I}}$, they contribute *coherently*, giving a factor $\propto (\sum_i m_i)^2$;
2. When the particles are at distance larger than the noise correlation lengths $r_C^{\text{R}}, r_C^{\text{I}}$, they contribute *incoherently* giving a factor $\propto \sum_i m_i^2$.

Because of these two properties, a reasonable estimate of the amplification factor in Eq. (40), is provided by Adler's formula [30, 31]:

$$A^\beta = A^\beta(r_C^\beta) = N^\beta(n^\beta m_0)^2 \quad \text{with} \quad \beta = \text{R, I}, \quad (42)$$

where A^β refers to A in the second line of Eq. (40) for $\beta = \text{R}$ and to A in the third line for $\beta = \text{I}$; n^β is the number of nucleons of mass m_0 inside a sphere of radius r_C^β , while N^β denotes the number of such spheres necessary for covering the entire object.

V. EXPERIMENTAL BOUNDS ON THE GRAVITATIONAL NOISE SPECTRUM

Discussing the experimental constraints on the noise correlator in its full generality is too difficult. We will limit the discussion to a restricted class of gaussian correlations functions, in such a way that the collapse dynamics is controlled by only two parameters (for a class of correlation function that leads to a HPZ type master equation—see Appendix C).

Specifically, we consider the Markovian limit by imposing

$$\tilde{D}^{\text{R}}(\mathbf{Q}, s) \approx \tilde{D}^{\text{R}}(\mathbf{Q}) \tau_0 \delta(s) \quad (43)$$

with $[\tau_0] = [T]$ (see Eq. (26)). From the definition of $D_{ij}(t, \tau)$ in Eq. (5), using the definition of the Fourier transform and Eq. (43), it is straightforward to show that $\tilde{D}^{\text{I}}(\mathbf{Q}) = 0$. In addition, to make contact with existing phenomenology for the CSL model [10], we assume that $\tilde{D}^{\text{R}}(\mathbf{Q})$ has the following correlation function:

$$\tilde{D}^{\text{R}}(\mathbf{Q}) = r_C^3 \exp(-r_C^2 \mathbf{Q}^2 / \hbar^2), \quad (44)$$

where $[r_C] = [L]$. With these assumptions, after some algebra, Eq. (39) reduces to

$$\partial_t \rho_t^{\text{CM}} = -\frac{i}{\hbar} [\hat{H}_0^{\text{CM}}, \rho_t^{\text{CM}}] - \frac{\xi^2 c^4 r_C^3 \tau_0}{(2\pi\hbar)^3 2\hbar^2} \int d\mathbf{Q} A(\mathbf{Q}) \exp(-r_C^2 \mathbf{Q}^2 / \hbar^2) \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \left[e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \rho_t^{\text{CM}} \right] \right]. \quad (45)$$

This equation should be compared with the CSL master equation [10]:

$$\partial_t \rho_t^{\text{CM}} = -\frac{i}{\hbar} [\hat{H}_0^{\text{CM}}, \rho_t^{\text{CM}}] - \frac{\lambda (4\pi r_C^2)^{3/2}}{(2\pi\hbar)^3} \int d\mathbf{Q} \frac{A(\mathbf{Q})}{m_0^2} \exp(-r_C^2 \mathbf{Q}^2 / \hbar^2) \left[e^{-\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \left[e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}}, \rho_t^{\text{CM}} \right] \right] \quad (46)$$

In particular, Eq. (45) reduces to the CSL master equation given in Eq. (46) by setting:

$$\xi = \frac{4\hbar\pi^{3/4}}{m_0 c^2} \sqrt{\frac{\lambda}{\tau_0}} \quad (47)$$

Still, the model has too many degrees of freedom: the magnitude ξ of the metric fluctuations, the time cutoff τ_0 and the space cutoff r_C . To simplify the discussion, we assume the time cutoff to be related to the space cutoff via $\tau_0 = r_C/c$. We can now set bounds on ξ by using the bounds already set for the CSL parameters λ and r_C . We have summarized the most recent bounds in Fig. 1.

We compare these results with the recent discovery of gravitational waves [...], observed in frequency range from 35 to 250 Hz and with a peak strain of 1.0×10^{-21} . Clearly, gravitational waves are real, while here the claim is that the collapse is caused by complex fluctuations of the metric. Also, gravitational waves typically have long wave lengths, while here the relevant part of the spectrum is at high frequencies (Fig. 1). However it is interesting to see that the order of magnitude of real waves and complex fluctuations—which allow for an efficient collapse and are compatible with experimental data—are not so far away from each other.

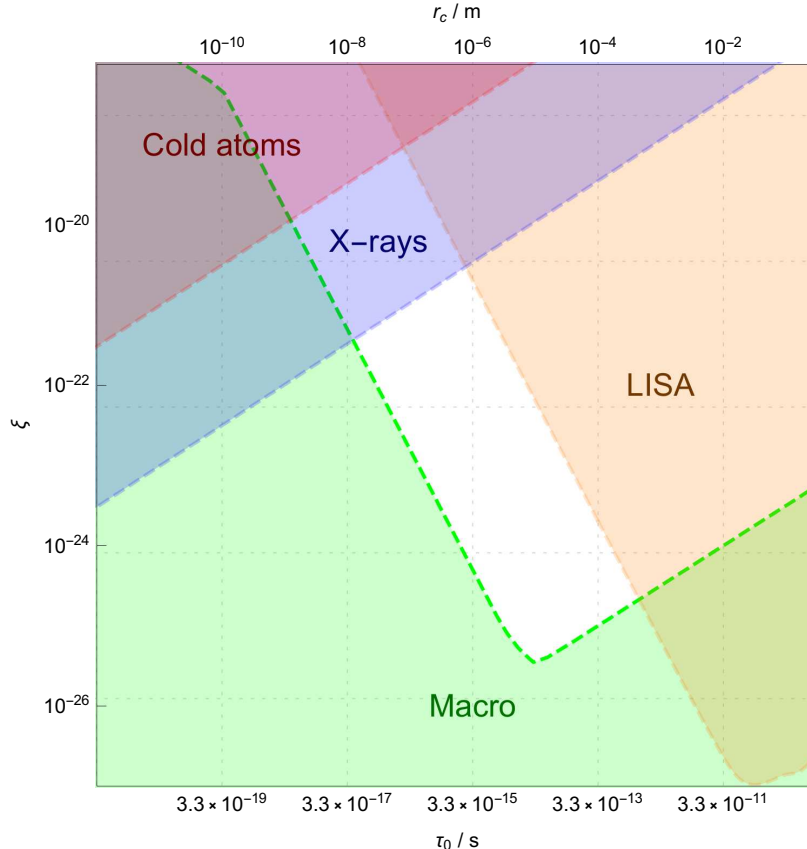


FIG. 1: (ξ, r_c) or equivalently (ξ, τ_0) parameter diagram of the gravity-induced collapse model. The white area is the allowed region. The blue shaded region (*X-rays*) is excluded by data analysis of X-rays measurements [32]. The orange shaded region (*LISA*) is excluded from data analysis of LISA Pathfinder [33]. The green shaded region (*Macro*) is an estimate of the region excluded by the requirement that the collapse is strong enough to localize macroscopic objects [30, 34]. Note that X-ray measurements sample the high frequency region of the spectrum ($\sim 10^{18}$ Hz) and would disappear if the noise correlator has a cutoff below such frequencies, which is plausible. In such a case, the stronger upper bound on the left part of the plane is given by data analysis with cold atom experiments (*Cold atoms*) [35].

VI. DISCUSSION AND CONCLUSIONS

Gravity related models of spontaneous wave function collapse are not new in the literature. We mention two of them. The Diosi-Penrose (DP) model [25–28] has the same structure as the model here considered, with two important differences: i. The noise is real and white in time; ii. The spatial correlation function is proportional to $G/|\mathbf{x} - \mathbf{y}|$. Although the model is certainly appealing in many ways, we see no reason why the noise correlator should have such a special form. Typically noises have rather complicated correlation functions, which have little or no connection with the form of the interaction.

The Schrödinger-Newton (SN) equation [28, 36, 37] descends from semi-classical gravity [38, 39] and contains a gravitational self-interaction term, which tends to suppress superpositions in space. However, as discussed in [40], this equation is not of the collapse-model type, in particular it is not capable of predicting the collapse of the wave function in space with the correct quantum probabilities.

In this paper we have investigated a novel proposal, where the collapse mechanism is driven by a complex fluctuating metric, as first suggested by Adler [1]. The correlation function should have

a non negligible contribution also from relatively high frequency components ($\sim 10^{15}$ Hz), contrary to current search for gravitational waves, which is limited to much lower frequencies.

By imposing the condition of no superluminal signaling, we derived the structure of the equation describing the evolution of the state vector (Eq. (18)), perturbatively up to the second order in ξ , the coupling constant which sets the magnitude of the gravitational noise. Then we proved that Eq. (18) defines a good collapse dynamics: it collapses the state vector to the eigenstates of the preferred basis (in our case, the position basis) and it has an amplification mechanism which guarantees that, even for small ξ , collapse effects becomes relevant for macroscopic objects.

In the last section we discussed experimental bounds on the parameters of the model. Interestingly enough the magnitude of the complex fluctuations needed for the collapse to be compatible with experimental data, and to guarantee the localization of macroscopic objects, is compatible with recent findings on gravitational waves. Although the supposed collapse is driven by the complex components of the metric, while gravitational waves have real components, and the relevant frequency ranges are different, one might expect that real and complex fluctuations interact and therefore should not be entirely different from each other.

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Appendix A: Justification of the collapse equation

We present the procedure outlined in the introduction, to justify the collapse equation. Here, to keep the notation simple, we focus on the case with only one operator A and one noise w_t . However, the generalization to the model described in Eq. (2) can be trivially done, since the noises are independent.

Let us consider the Hamiltonian $\hat{H} = \hat{H}_0 + i\hbar(\sqrt{\lambda}\hat{A}w_t + \hat{O})$ or, in the Itô language, the stochastic differential equation:

$$d\phi_t = \left[-i\hat{H}_0 dt + \sqrt{\lambda}\hat{A}dw_t + \hat{O} \right] \phi_t; \quad (\text{A1})$$

throughout this section, we set $\hbar = 1$. We will fix the form of \hat{O} by requiring no superluminal signaling.

The norm of ϕ_t is not conserved. In order to write the equation for the normalized vector $\psi_t = \phi_t / \|\phi_t\|$, let us consider the process $N_t = \langle \phi_t | \phi_t \rangle$. Using Itô rules ($dN_t = \langle d\phi_t | \phi_t \rangle + \langle \phi_t | d\phi_t \rangle + \langle d\phi_t | d\phi_t \rangle$) one proves that it satisfies the stochastic differential equation:

$$dN_t = \left[2\sqrt{\lambda}\langle \hat{A} \rangle_t dw_t + \lambda\langle \hat{A}^2 \rangle_t dt + \langle (\hat{O}^\dagger + \hat{O}) \rangle_t dt \right] N_t, \quad (\text{A2})$$

where we have defined $\langle \hat{A} \rangle_t = \langle \phi_t | \hat{A} | \phi_t \rangle / \|\phi_t\|^2 = \langle \psi_t | \hat{A} | \psi_t \rangle$, and similarly for all other operators. From this one can derive the equation for $N_t^{-1/2}$:

$$dN_t^{-1/2} = \left[-\sqrt{\lambda}\langle \hat{A} \rangle_t dw_t + \frac{3}{2}\lambda\langle \hat{A}^2 \rangle_t dt - \frac{1}{2}\lambda\langle \hat{A}^2 \rangle_t dt - \frac{1}{2}\langle (\hat{O}^\dagger + \hat{O}) \rangle_t dt \right] N_t^{-1/2}, \quad (\text{A3})$$

and next the equation for $\psi_t = \phi_t N_t^{-1/2}$:

$$d\psi_t = \left[-i\hat{H}_0 dt + \sqrt{\lambda}(\hat{A} - \langle \hat{A} \rangle_t) dW_t + \lambda \left(\frac{3}{2} \langle \hat{A} \rangle_t^2 - \frac{1}{2} \langle \hat{A}^2 \rangle_t - \hat{A} \langle \hat{A} \rangle_t \right) dt + \left(\hat{O} - \frac{1}{2} \langle (\hat{O}^\dagger + \hat{O}) \rangle_t \right) dt \right] \psi_t. \quad (\text{A4})$$

As we can see, the normalized vector evolves according to a nonlinear stochastic dynamics. The stochastic ensemble of pure states $\rho_t^W = |\psi_t\rangle\langle\psi_t|$ obeys the following dynamics:

$$d\rho_t^W = -i[H, \rho_t^W] + \lambda \left(4\langle \hat{A} \rangle_t^2 \rho_t^W - \langle \hat{A}^2 \rangle_t \rho_t^W - 2\hat{A} \langle \hat{A} \rangle_t \rho_t^W - 2\rho_t^W \hat{A} \langle \hat{A} \rangle_t - \hat{A} \rho_t^W \hat{A} \right) dt + \left(\hat{O}^\dagger \rho_t^W + \rho_t^W \hat{O} - \langle (\hat{O}^\dagger + \hat{O}) \rangle_t \rho_t^W \right) dt + (\text{extra terms}) dW_t. \quad (\text{A5})$$

When taking the expectation value to compute the dynamics for the density matrix $\rho_t = \mathbb{E}[\rho_t^W]$, the ‘extra terms’ average to 0, while the remaining terms generate a nonlinear evolution for the ensemble. This can be avoided by choosing $O = -(\lambda/2)\hat{A}^2 + 2\lambda(\hat{A} - \langle \hat{A} \rangle_t)\langle \hat{A} \rangle_t$, in which case all nonlinear terms cancel, and the equation for ρ_t becomes of the Lindblad type:

$$\frac{d}{dt}\rho_t = -i[\hat{H}_0, \rho_t] - \frac{\lambda}{2}[\hat{A}, [\hat{A}, \rho_t]]; \quad (\text{A6})$$

in turn, Eq. (A1) reduces to Eq. (1). This completes the argument.

Appendix B: Non-relativistic coupling between a gravitational background and the local mass density

The action of a matter field in curved space is described by:

$$S = \int d^4x \sqrt{-g} \mathcal{L}_m \quad (\text{B1})$$

where \mathcal{L}_m is the matter Lagrangian, $g_{\mu\nu}$ is the metric tensor and $\sqrt{-g} = \sqrt{-\det[g_{\mu\nu}]}$. We consider a perturbation $h_{\mu\nu}$ around the flat metric $\eta_{\mu\nu}$, and we Taylor expand the action around it:

$$S = \int d^4x \left[\mathcal{L}_m^{(0)} + \frac{1}{\sqrt{-\eta}} \frac{\partial(\sqrt{-g} \mathcal{L})}{\partial g_{\mu\nu}} \Big|_{\eta_{\mu\nu}} h_{\mu\nu} \right] + \mathcal{O}(h^{\mu\nu} h^{\delta\sigma}); \quad (\text{B2})$$

the apex (0) denotes the quantities in the flat space-time $\eta_{\mu\nu}$. The stress energy tensor associated to the Lagrangian \mathcal{L}_m is defined as follows [41]:

$$T^{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\partial(\sqrt{-g} \mathcal{L}_m)}{\partial g_{\mu\nu}}, \quad (\text{B3})$$

and Eq. (B2) can be rewritten in the form

$$S = \int d^4x \left[\mathcal{L}_m^{(0)} - \frac{1}{2} h^{\mu\nu} T_{\mu\nu}^{(0)} \right], \quad (\text{B4})$$

where from now on we neglect higher order terms. In the weak field limit, gravity couples to matter through the stress energy tensor.

We now derive the non relativistic limit, for a Klein-Gordon Lagrangian:

$$\mathcal{L}_m^{(0)} = \frac{-\hbar^2}{2m} \left(\eta^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - \frac{(mc)^2}{\hbar^2} \psi^* \psi \right). \quad (\text{B5})$$

The interacting Lagrangian becomes:

$$\mathcal{L}_{int}^{(0)} = -\frac{1}{2} h^{\mu\nu} T_{\mu\nu}^{(0)} = \frac{\hbar}{2m} \left(\partial_\mu \psi^* \partial_\nu \psi (h^{\mu\nu} - h_\rho^\rho \eta^{\mu\nu}) - h_\rho^\rho \frac{mc^2}{2\hbar^2} \psi^* \psi \right) \quad (\text{B6})$$

The non-relativistic limit can be obtained by rewriting the relativistic wave function as follows:

$$\psi(x) = e^{\frac{i}{\hbar} mc x_0} \varphi(x), \quad (\text{B7})$$

and assuming that the following relation holds:

$$\left| \frac{mc}{\hbar} \varphi \right| \gg |i\partial_\mu \varphi|, \quad (\text{B8})$$

meaning that the rest energy associated to the field φ is much bigger than the momentum energy. Inserting Eq. (B7) in Eqs. (B5) and (B6) one obtains

$$\mathcal{L}_m^{(0)} = -\frac{\hbar}{2m} \left[\partial_0 \varphi^* \partial_0 \varphi + i \frac{mc}{\hbar} (\varphi^* \partial_0 \varphi - (\partial_0 \varphi^*) \varphi) + \partial_i \varphi^* \partial_i \varphi \right] \quad (\text{B9})$$

and

$$\begin{aligned} \mathcal{L}_{int}^{(0)} = -\frac{1}{2} h^{\mu\nu} T_{\mu\nu}^{(0)} = & -\frac{\hbar^2}{2m} \left[h^{00} \left(\partial_0 - i \frac{mc}{\hbar} \right) \varphi^* \left(\partial_0 + i \frac{mc}{\hbar} \right) \varphi \right. \\ & - h^{0i} \left(\left(\partial_0 - i \frac{mc}{\hbar} \right) \varphi^* \partial_i \varphi + \partial_i \varphi^* \left(\partial_0 + i \frac{mc}{\hbar} \right) \varphi \right) \\ & \left. + (h^{ij} - h_\rho^\rho \eta^{ij}) \partial_i \varphi^* \partial_j \varphi \right]. \end{aligned} \quad (\text{B10})$$

Under the assumption in (B8), we arrive at the symmetrized free Schrödinger Lagrangian ($x_0 = ct$):

$$\mathcal{L}_m^{(0)} \simeq \frac{i\hbar}{2} (\varphi^* \partial_t \varphi - \partial_t (\varphi^*) \varphi) + \frac{\hbar^2}{2m} \partial_i \varphi^* \partial^i \varphi \quad (\text{B11})$$

and the interaction Lagrangian

$$\mathcal{L}_{int}^{(0)} = -\frac{mc^2}{2} h^{00} \varphi^* \varphi. \quad (\text{B12})$$

The conjugate momenta associated to the total Lagrangian $\mathcal{L} = \mathcal{L}_m^{(0)} + \mathcal{L}_{int}^{(0)}$ are

$$\begin{aligned} \pi &= \frac{\partial \mathcal{L}}{\partial (\partial_t \varphi)} = \frac{i\hbar}{2} \varphi^*, \\ \pi^* &= \frac{\partial \mathcal{L}}{\partial (\partial_t \varphi^*)} = -\frac{i\hbar}{2} \varphi, \end{aligned} \quad (\text{B13})$$

and the Hamiltonian density is:

$$\mathcal{H}(x) = \pi \partial_t \varphi^* + \pi^* \partial_t \varphi - \mathcal{L} = \frac{\hbar^2}{2m} \partial_i \varphi^* \partial^i \varphi + \frac{mc^2}{2} h^{00} \varphi^* \varphi \quad (\text{B14})$$

leading, after integration by parts, to the Hamiltonian

$$H = \int d^3x \varphi^*(\mathbf{x}, t) \left(-\frac{\hbar^2}{2m} \partial_i \partial^i + \frac{mc^2}{2} h^{00}(\mathbf{x}, t) \right) \varphi(\mathbf{x}, t) \quad (\text{B15})$$

Promoting the field $\varphi(x)$ ($\varphi^*(x)$) and it's conjugate momenta $\pi(x)$ ($\pi(x)^*$) to operator:

$$\begin{aligned} \varphi(\mathbf{x}, t) &\rightarrow \hat{\varphi}(\mathbf{x}, t), \\ \pi(\mathbf{x}, t) &\rightarrow \hat{\pi}(\mathbf{x}, t) \end{aligned} \quad (\text{B16})$$

and imposing the canonical quantization rule, *i.e.*

$$[\hat{\varphi}(\mathbf{x}, t), \hat{\pi}(\mathbf{y}, t)] = [\hat{\varphi}^\dagger(\mathbf{x}, t), \hat{\pi}^\dagger(\mathbf{y}, t)] = i\hbar \delta(\mathbf{x} - \mathbf{y}) \quad (\text{B17})$$

one obtains the hamiltonian

$$\hat{H} = \int d^3x \hat{\varphi}^\dagger(\mathbf{x}) H_1(x) \hat{\varphi}(\mathbf{x}) \quad (\text{B18})$$

where

$$H_1(\mathbf{x}, t) = -\frac{\hbar^2}{2m} \partial_i \partial^i + \frac{mc^2}{2} h^{00}(\mathbf{x}, t) \quad (\text{B19})$$

is the single particle hamiltonian expessed in the position basis.

Appendix C: Relation with the HPZ master equation

Let us start with the center of mass master equation given by Eq. (39) with the free particle Hamiltonian $\hat{H}_0^{\text{CM}} = \frac{\hat{\mathbf{P}}^2}{2m}$, where $\hat{\mathbf{P}}$ is the center of mass momentum operator. We make two assumptions on the noise correlation functions $\tilde{D}^{\text{R}}(\mathbf{Q}, s)$, $\tilde{D}^{\text{I}}(\mathbf{Q}, s)$ and on the center of mass state ρ_t . Loosely speaking, we restrict to a nearly Markovian regime and assume that the exchanged momentum between noise and system is small. Mathematically, we give the sufficient conditions to expand the operators to quadratic order, *i.e.* to order $\mathcal{O}(\hat{X}^2)$, $\mathcal{O}(\hat{P}^2)$, $\mathcal{O}(\hat{X}\hat{P})$:

- (a) The noise correlations times are small and the state ρ_t is such that:

$$e^{\frac{i}{\hbar} \hat{H}_0^{\text{CM}} s} \approx 1 + \frac{i}{\hbar} \hat{H}_0^{\text{CM}} s = 1 + \frac{i}{\hbar} \frac{\hat{P}^2}{2m} s. \quad (\text{C1})$$

- (b) The noise momenta correlations are small and the state ρ_t is such that:

$$e^{\frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}}} \approx 1 + \frac{i}{\hbar} \mathbf{Q} \cdot \hat{\mathbf{X}} - \frac{1}{\hbar^2} (\mathbf{Q} \cdot \hat{\mathbf{X}})^2 \quad (\text{C2})$$

Moreover, the noise momenta correlations depend only the modulus $Q = |\mathbf{Q}|$:

$$\tilde{D}^{\text{R}}(\mathbf{Q}, s) = \tilde{D}^{\text{R}}(Q, s), \quad \tilde{D}^{\text{I}}(\mathbf{Q}, s) = \tilde{D}^{\text{I}}(Q, s). \quad (\text{C3})$$

which is equivalent, as follows from Eq. (31), to assuming a noise correlation isotropic in space $D^{\text{R}}(\mathbf{r}, s) = D^{\text{R}}(r, s)$ and $D^{\text{I}}(\mathbf{r}, s) = D^{\text{I}}(r, s)$ with $r = |\mathbf{r}|$.

- (c) The noise correlation time τ_0 is small with respect to the evolution time t .

Applying the above assumptions (a), (b) and (c), the formula in Eq. (42) for the amplification factors and using the identity

$$\int d\mathbf{Q} f(Q)(\mathbf{Q} \cdot \mathbf{X})(\mathbf{Q} \cdot \mathbf{Y}) = \left(\int_0^\infty dQ f(Q) Q^4 \right) \frac{4\pi}{3} \mathbf{X} \cdot \mathbf{Y}, \quad (\text{C4})$$

where $f(Q)$ denotes a generic function, we can perform the \mathbf{Q} integration in Eq. (39). After some algebra we obtain the simplified master equation:

$$\begin{aligned} \frac{d\rho_t^{\text{CM}}}{dt} = & -\frac{i}{\hbar} \sum_{j=1}^3 \left[\frac{\hat{P}_j^2}{2m}, \rho_t \right] - \eta \frac{A^{\text{R}}(r_C^{\text{R}})}{m_0^2} \sum_{j=1}^3 \left[\hat{X}_j, [\hat{X}_j, \rho_t] \right] \\ & + \Pi \frac{A^{\text{R}}(r_C^{\text{R}})}{m_0^2} \sum_{j=1}^3 \left[\hat{X}_j, [\hat{P}_j/m, \hat{\rho}] \right] - i\Upsilon \frac{A^{\text{I}}(r_C^{\text{I}})}{m_0^2} \sum_{j=1}^3 \left[\hat{X}_j, \left\{ \hat{P}_j/m, \rho_t \right\} \right], \end{aligned} \quad (\text{C5})$$

where

$$\eta = \frac{m_0^2 c^4 \xi^2}{6\pi^2 \hbar^7} \int_0^\infty dQ \int_0^\infty d\tau \tilde{D}^{\text{R}}(Q, \tau) Q^4, \quad (\text{C6})$$

$$\Pi = \frac{m_0^2 c^4 \xi^2}{6\pi^2 \hbar^7} \int_0^\infty dQ \int_0^\infty d\tau \tau \tilde{D}^{\text{R}}(Q, \tau) Q^4, \quad (\text{C7})$$

$$\Upsilon = -\frac{m_0^2 c^4 \xi^2}{6\pi^2 \hbar^7} \int_0^\infty dQ \int_0^\infty d\tau \tau \tilde{D}^{\text{I}}(Q, \tau) Q^4, \quad (\text{C8})$$

are three phenomenological parameters (given assumption (c) these do not depend on t), while $A^{\text{R}}(r_C^{\text{R}})/m_0^2$, $A^{\text{I}}(r_C^{\text{I}})/m_0^2$ are dimensionless amplification factors, related to the size and shape of the composite object as well as with the noise spatial correlation cutoffs r_C^{R} and r_C^{I} (see Sec. IV).

Eq. (C5) has the same structure as the HPZ master equation [42], except for the absence of the HPZ term that breaks translational invariance. The reason why the HPZ master equation breaks translational invariance lies in its founding assumption: a particle in a harmonic potential coupled to a bath of oscillators. In our case, loosely speaking, the external oscillators correspond to the complex noise, while the harmonic potential, which explicitly breaks translational invariance, is absent. Our noise does not break translational invariance, as we have assumed that the correlation function is explicitly translationally invariant (see Sec. IV).

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- [43] here the discrete label “ j ” is substitute by continuous label \mathbf{x} representing the space points. Accordingly the discrete sums over j becomes integrals over \mathbf{x}
- [44] In the expansion of \hat{O} we do not include a zeroth order term i.e. one independent from ξ . This because \hat{O} is introduced to add corrective terms due to the presence of the noise terms, and therefore is expected to be zero in the limit $\xi \rightarrow 0$.
- [45] see pag.9 [29]
- [46] Here again the discrete label i is substituted by the continuous parameter \mathbf{x} .
- [47] To be precise, also for a rigid body, together with the 3 degrees of freedom of the center of mass, one should consider also the 3 degrees of freedom describing rotations of the whole system. These rotations are into the relative. However very hard to take into account and it is not expect to change too much. Therefore we will not consider this